JOURNAL OF MATERIALS SCIENCE 8 (1973) 1065-1070

Temperature and concentration dependence of the critical resolved shear stress of cadmium-zinc alloy single crystals

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The increase of the critical resolved shear stress of cadmium single crystals by additions of zinc has been investigated in the temperature range 77 to 295K. The temperature dependence of the critical resolved shear stress can be divided into two temperature regions. At all temperatures the critical resolved shear stress was found to increase with $c^{2/3}$ where c is the atomic concentration of zinc as solute. The concentration dependence of the plateau stress is explained according to the theory of Labusch [5].

1. Introduction

Considerable attention has been given recently to the effect of solute atoms on the mechanical properties of metals. A review of possible mechanisms responsible for solid solution hardening, i.e., for the increase of the critical resolved shear stress in the temperature-independent range (so-called plateau stress – τ_p) with the solute concentration, has been given by Haasen [1-3].

Fleischer [4] has shown that the solid solution hardening can be explained if the interaction due both to the size effect and to the modulus effect are taken into account. He found that

$$\tau_p = Z_F\, G\epsilon_F^{3/2}\, c^{1/2} + \tau_0\,(c = 0) \tag{1}$$

where

$$\epsilon_F = |\, \eta' - \alpha\delta \,| \tag{2}$$

$$\eta' = \frac{\eta}{1 + \frac{1}{2}|\,\eta\,|} \; . \tag{3}$$

δ is the size misfit parameter defined by the relative change of the lattice parameter (or Burgers vector $\boldsymbol{b}$) with atomic concentration of solute atoms, i.e.,

$$\delta = \frac{1}{\boldsymbol{b}}\frac{\mathrm{d}b}{\mathrm{d}c} \; . \tag{4}$$

η is the relative change of the shear modulus G with alloying given by

$$\eta = \frac{1}{G}\frac{\mathrm{d}G}{\mathrm{d}c} \; . \tag{5}$$

$\tau_0\,(c = 0)$ is the critical resolved shear stress of the pure metal, $Z_F = 1/760$ and $\alpha = 3$ for screw dislocations and $\alpha = 16$ for edge dislocations.

Recently Labusch [5] has used a more rigorous statistical treatment for the calculation of the critical resolved shear stress for a movement of dislocations through randomly distributed solute atoms as obstacles. According to Labusch [5]

$$\tau_p = Z_L\, G\epsilon_L^{4/3}\, c^{2/3} + \tau_0\,(c = 0) \tag{6}$$

where

$$\epsilon_L = (\eta'^2 + \alpha^2\delta^2)^{1/2} \tag{7}$$

and

$$Z_L = 1/750\,.$$

Experimental results on solid solution hardening in fcc single crystals were critically summarized by Jax *et al* [6]. They have shown that the solid solution hardening in fcc single crystals (namely in gold, silver and copper alloys) is described by Equation 6 better than by Equation 1.

However, little information is available on solid solution hardening in hcp alloys. Lately solid solution hardening has been investigated in single crystals of Mg base alloys [7-9]. In the present work, the effect of zinc as solute on the critical resolved shear stress of cadmium single crystals is studied.

2. Experimental procedure

Polycrystalline rods of Cd-Zn alloys were prepared from high purity cadmium (99999%) and high purity zinc (99999%). The single

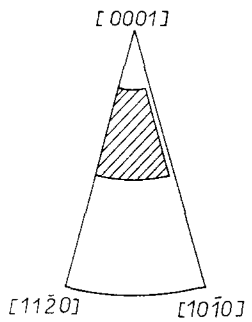

Figure 1 Orientation of the deformed samples.

crystals (20 to 25 cm in length and 4 mm in diameter) were grown from the melt in a glass tube using a modified Bridgman method [10]. The samples having 5 cm gauge length were annealed at 180°C for 6 h and then cooled to room temperature. Specimens with the orientation allowing only basal slip were used (Fig. 1).

The single crystals were deformed at a constant cross-head speed which gave an initial shear strain-rate of 1.2 to 1.7 $\times$ 10^{-4} sec^{-1}. The tensile tests were performed at temperatures between 77 and 295K. The temperature did not change more than $\pm$ 2°C during each experiment.

3. Experimental results

The resolved shear stress—shear strain curves for Cd-Zn alloy single crystals deformed at 77K are shown in Fig. 2. The shear stress—shear strain curves exhibit the linear stage A (easy glide). This allows one to define the critical resolved shear stress (CRSS) as the value of τ extrapolated to a = 0. It is obvious that the CRSS increases with increasing concentration of zinc.

Fig. 3 shows the shear stress—shear strain curves of cadmium single crystals with 0.38 at. % zinc deformed at various temperatures. The temperature dependence of the CRSS for Cd-Zn alloys is shown in Fig. 4. τ_0 decreases with increasing temperature up to 260K. For $T >$ 260K, τ_0 is independent of temperature; a so-called plateau stress, τ_p, is reached. The decrease of τ_0 with increasing temperature (for $T <$ 260K) is linear, except for the value at 77K.

Fig. 5 shows the dependence of τ_0 on the atomic concentration of zinc, c. The values of τ_0 at 0K have been obtained by extrapolation of the curves in Fig. 4. The data obtained by this procedure are practically the same as measured by Lukáč and Will [11] at liquid helium temperature (the points are marked as □ in Fig. 4). From Fig. 5 it follows that the CRSS is not a linear function of the solute concentration neither does it depend linearly on $c^{1/2}$.

If the CRSS is plotted against $c^{2/3}$, a linear relationship is obtained at all temperatures and for the whole concentration range studied (Fig. 6).

Table I gives the values of the plateau stress

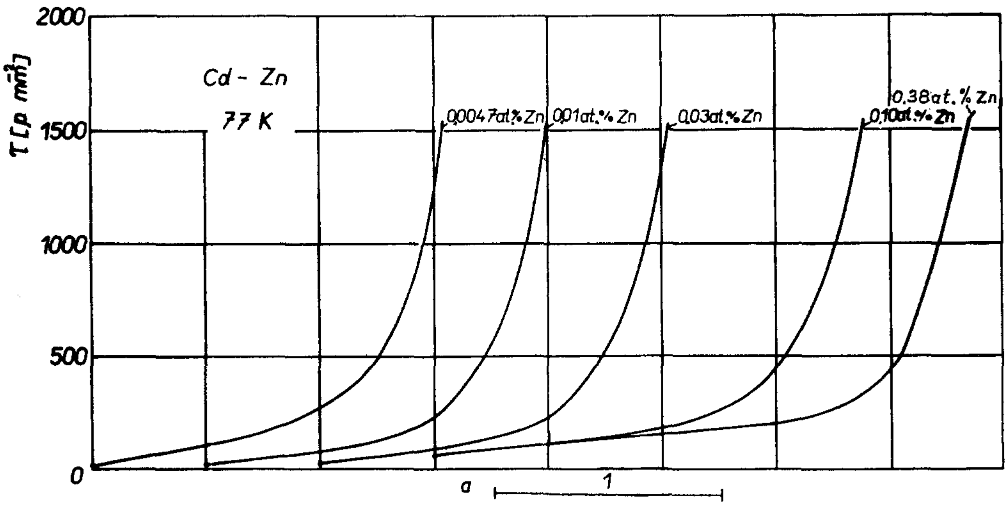

Figure 2 Resolved shear stress—shear strain curves for Cd-Zn alloy single crystals deformed at 77K. So as to avoid overlap, the curves are shifted by the shear strain a = 0.5 with respect to one another.

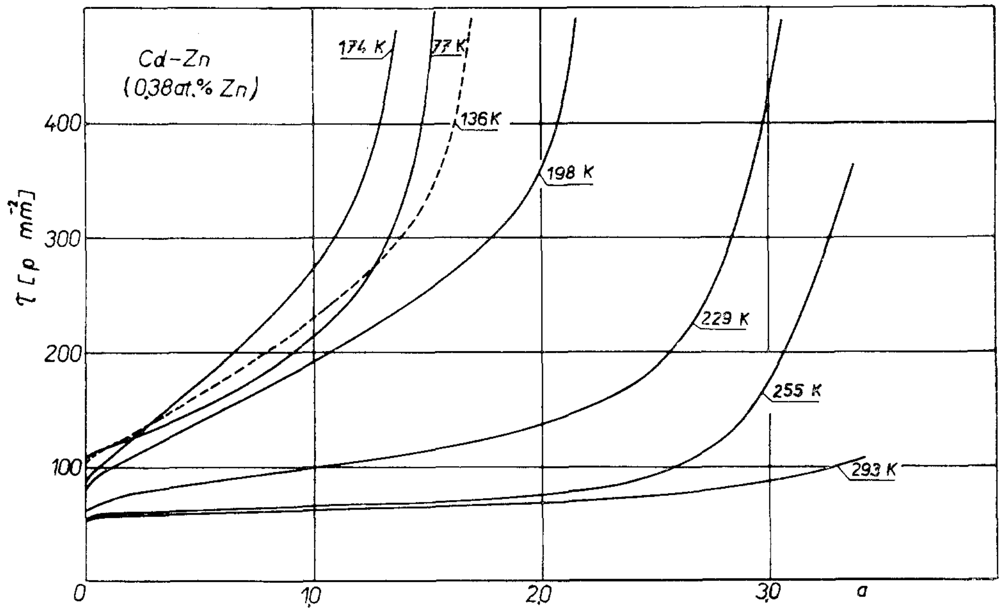

Figure 3 Resolved shear stress—shear strain curves of cadmium single crystals with 0.38 at. % Zn, deformed at various temperatures.

TABLE I

Concentration of Zn (at. %)	τ_p (p mm^{-2})	$\tau_0(0,c)$ (p mm^{-2})
0.0047	11	20
0.010	13	27
0.030	16.5	38
0.052	20	—
0.10	27	75
0.38	57	165

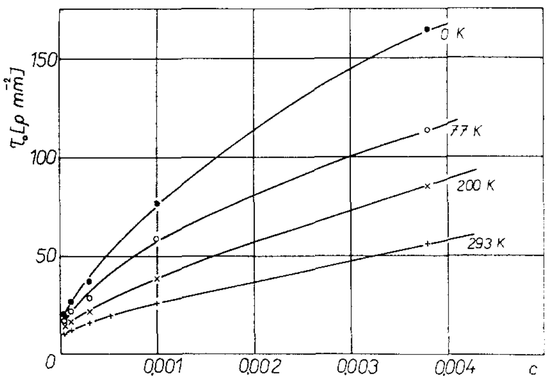

Figure 5 The increase of the CRSS with the atomic concentration of zinc.

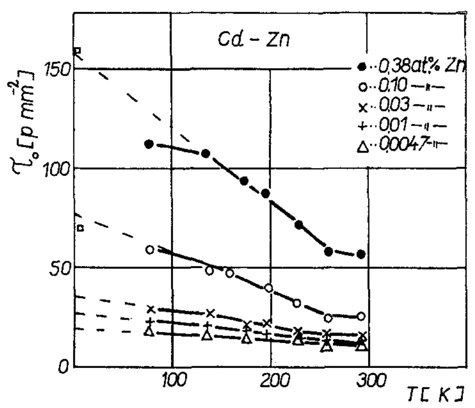

Figure 4 Temperature dependence of the critical resolved shear stress for Cd-Zn alloy single crystals. (The points marked as □ were measured by Lukáč and Will [11].)

τ_p as well as the extrapolated CRSS at 0K, $\tau_0(o,c)$, for Cd-Zn single crystals.

4. Discussion

The concentration dependence of the critical resolved shear stress will be discussed separately for (a) the plateau stress, (b) the low temperature region.

4.1. Plateau stress

Both the Suzuki effect [12] and the short range order mechanism [13] can be ruled out because

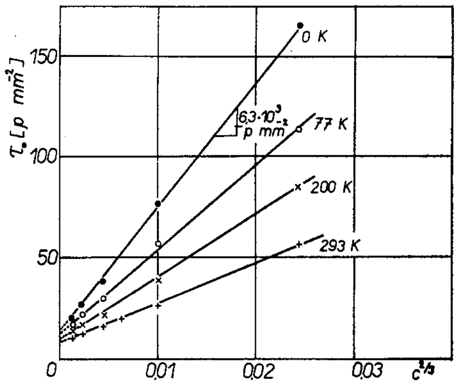

Figure 6 Critical resolved shear stress dependent on $c^{2/3}$.

both predict a concentration dependence of τ_0 different from that observed here. Our results on cadmium-zinc alloys indicate that the τ_p – $c^{2/3}$ dependence fits better than the τ_p – $c^{1/2}$ one (Fig. 7a). This means that the concentration dependence of the plateau stress is described by the Equation 6 better than by Equation 1. To prove the validity of Equations 1 and 6, the parameter η must be known. This has not been measured for Cd-Zn alloys. Its approximate value can be estimated by means of Relation [4, 14]

$$\eta \approx \frac{2(G_1 - G)}{G_1 + G} \tag{8}$$

where G and G_1 are the shear moduli of solvent and solute elements, respectively.

If η is determined using Relation 8 then the value of Z_L calculated from the experiment is in good agreement with that predicted by Labusch [5] (see Table II).

It was also found for the Mg-Cd alloys [7] that τ_p follows a $c^{2/3}$ law (Fig. 7b). Akhtar and Teghtsoonian [8] have plotted the CRSS of Mg-Zn single crystals against the square root of the solute concentration and obtained the dependence having two parts with the different slopes. If their results are plotted against $c^{2/3}$ (Fig. 7c), a straight line results [11]. This again favours a $c^{2/3}$ dependence to a $c^{1/2}$ dependence.

Comparing the present results with those mentioned above for Mg-Cd [7] and Mg-Zn [8] alloy single crystals:

(i) it is obvious from Figs. 7a to c that the dependence of the plateau stress on the square root of the solute concentration is always to be divided into two linear parts;

(ii) on the other hand the τ_p versus $c^{2/3}$ dependence gives a single straight line in the whole investigated concentration range, which crosses the stress axis at the value close to the CRSS of pure metal;

(iii) it is possible to compare the values of Z_F and Z_L in Equation 1 and 6 respectively, with those obtained from the experiments, i.e., from the slopes ($d\tau_p/dc^{2/3}$) and ($d\tau_p/dc^{1/2}$) respectively (see Table II). In Table II ($d\tau_p/dc^{2/3}$) and ($d\tau_p dc^{1/2}$) are given, as well as the values of δ [15-17] and η . η has been measured only for Mg-Cd alloys [18]; for Mg-Zn and Cd-Zn it was calculated with the help of Equation 8 [19-21].

To conclude, the theory of Labusch [5] describes the experimental results better than the theory of Fleischer [4]. The value of α which fits the experimental data is 3. This means that screw dislocations are responsible for solid solution hardening in hexagonal alloys.

4.2. Low temperature region

Below 260 K, the temperature dependence of the CRSS can be given in the form

$$\tau_0(T,c) = \tau_0(0,c) - B\,T \tag{9}$$

where $\tau_0(0,c)$ and B depends on the concentration of zinc as shown in Figs. 6 and 8, respectively:

$$\tau_0(0,c) = \tau_0(0,0) + K_1\,c^{2/3} \tag{10}$$

and

TABLE II

System	$d\tau_p/dc^{1/2}$ (p mm^{-2})	$d\tau_p/dc^{2/3}$ (p mm^{-2})	$\lvert\delta\rvert$	$\lvert\eta'\rvert$	$1/Z_F$		$1/Z_L$	
					($\alpha = 3$)	($\alpha = 16$)	($\alpha = 3$)	($\alpha = 16$)
Cd-Zn	7.5×10^2	2×10^3	0.1	0.48	1900	8200	740	2200
Mg-Cd	6.5×10^2	1.4×10^3	0.07	0.33	900	4300	530	1800
Mg-Zn	2.1×10^3	3.9×10^3	0.2	0.59	1000	5600	520	2400

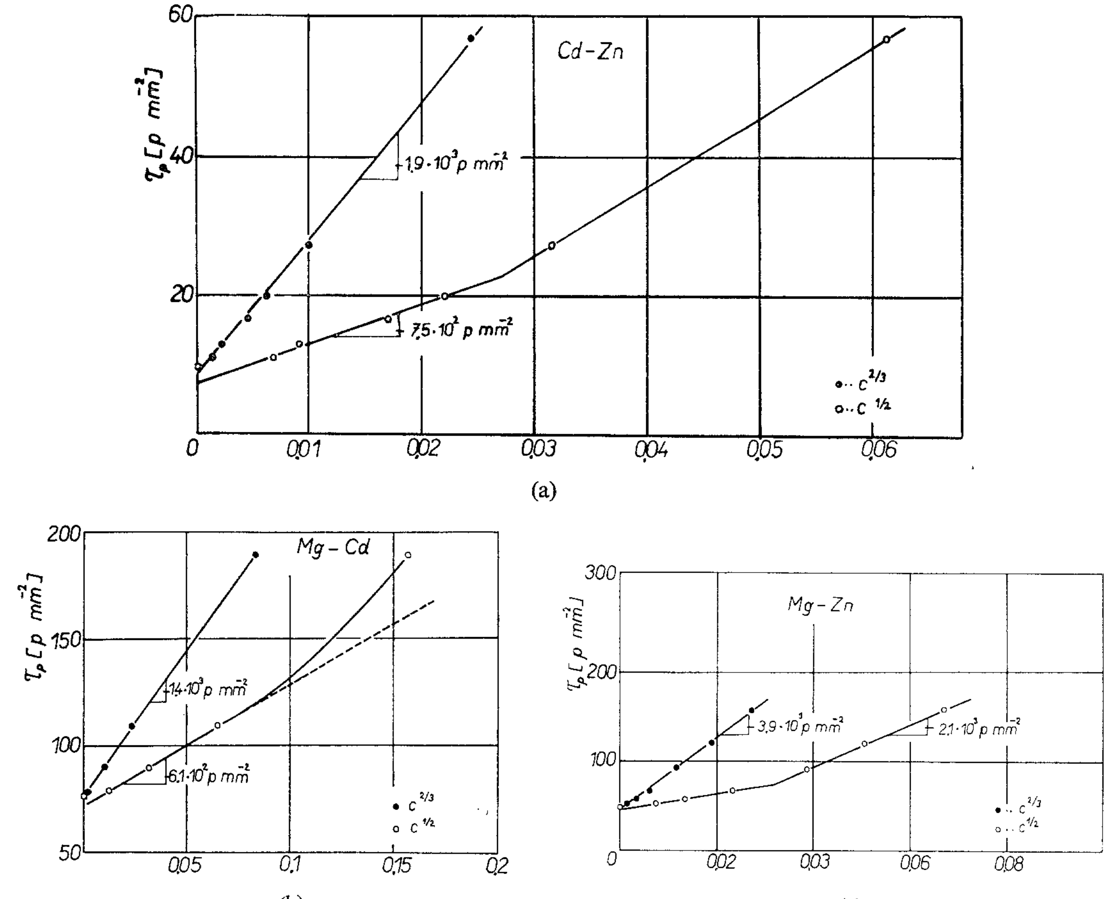

Figure 7 (a) Concentration dependence of the plateau stress for Cd-Zn single crystals (present results). (b) Concentration dependence of the plateau stress for Mg-Cd single crystals from [7]. (c) Concentration dependence of the plateau stress for Mg-Zn single crystals plotted by the present authors from results of Akhtar and Teghtsoonian [8].

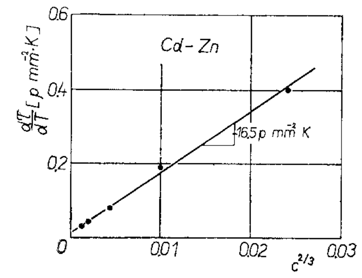

Figure 8 Concentration dependence of (d τ_0/dT) in low temperature region.

$$\frac{\mathrm{d}\tau_0(T,c)}{\mathrm{d}\,T} \equiv B = K_0 + K_2\,c^{2/3} \qquad (11)$$

where K_0, K_1 and K_2 are constants.

The pronounced temperature dependence of the CRSS is usually analysed in terms of thermally-activated glide of dislocations over local obstacles. Of the possible thermally activated processes the most probable is the unpinning of dislocations held by solute atoms. The concentration dependence of the CRSS at low temperatures is very often explained in accordance with Friedel's model [22], where dislocations are assumed to move in zig-zag form. According to Friedel [22], the CRSS should increase linearly with solute concentration, which is not observed in our experiment; neither is it in [11]. A nonlinear concentration dependence of the CRSS at low temperatures was also obtained for some Mg-base alloys [7-9].

Qualitatively the observed concentration

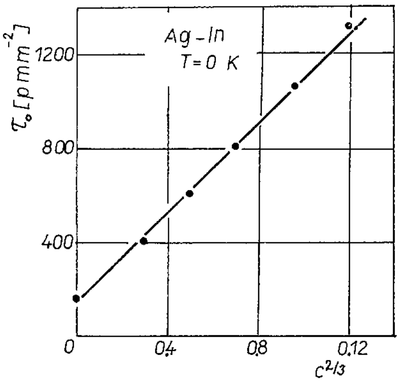

Figure 9 Critical resolved shear stress versus $c^{2/3}$ for Ag-In single crystals plotted by the present authors from results of Rogausch [25].

dependence of the CRSS – Equations 9 to 11 – can be explained on the basis of Labusch's statistical theory [5, 23]. According to this theory [23], the concentration dependence of the CRSS at 0K has a form very similar to Relation 6. He has also pointed out that when all theories of solid solution hardening are carried out correctly, then the $\tau_0(0,c)$ versus $c^{2/3}$ dependence holds [23]. More recently Labusch *et al* [24] have shown that the CRSS in the low temperature region should decrease with increasing temperature and increase linearly with $c^{2/3}$ at each temperature. Our results are in qualitative agreement with this theory.

In this connection it is interesting to note that Equation 10 holds not only for hexagonal alloys [7, 9, 11] but the same concentration dependence of the CRSS at 0K is valid for some fcc alloys. The results of Rogausch [25] can serve as one example. If the CRSS at 0K for Ag-In alloys are plotted against $c^{2/3}$, the straight line crosses the τ-axis near τ_0 for Ag (Fig. 9).

5. Conclusions

1. Solid solution hardening in cadmium single crystals containing Zn was investigated in the temperature region from 77K to room temperature.
2. In Cd-Zn alloy single crystals the temperature dependence of the CRSS can be divided into two temperature regions. At low temperatures, up to 260K, the CRSS decreases with temperature, while for $T > 260$K, it is temperature independent.
3. At all temperatures the CRSS depends linearly on $c^{2/3}$.
4. The concentration dependence of the CRSS can be explained by the theory of Labusch [5, 23].

Acknowledgement

The authors are very grateful to Dr P. Kratochvíl for many valuable comments.

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Received 21 September 1972 and accepted 14 February 1973.